Nonperturbative effects of the periodic potential on the field-induced spin-density wave and the sign of the quantum Hall effect in the quasi-one-dimensional conductor (TMTSF)$_2$ClO$_4$

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We show that in (TMTSF)$_2$ClO$_4$ the field-induced spin-density wave (FISDW) with negative quantum number ($N = -2$) of the nesting vector is stabilized in some region in the parameters of magnetic field and the strength of the anion potential, which corresponds to the very recently observed phase diagram of (TMTSF)$_2$ClO$_4$ in the parameter plane of magnetic field vs cooling rate by Matsunaga et al. [J. Phys. IV 131, 269 (2005)]. The spin-density wave is induced by the magnetic field in the quasi-one-dimensional conductors such as (TMTSF)$_2$PF$_6$ and (TMTSF)$_2$ClO$_4$. The wave vector of the FISDW is quantized and the Hall conductivity is quantized corresponding to the quantum number ($N$) of the wave vector. In (TMTSF)$_2$ClO$_4$, the ordering of the anion ClO$_4$ makes the periodic potential, which has been known to drastically affect the FISDW. We study the instability to the FISDW by taking the eigenstates in the magnetic field numerically, with the potential being treated nonperturbatively. We obtain the phase diagram of the quantum number $N$ for FISDW in the quasi-one-dimensional systems in the parameter plane of magnetic field and the strength of the periodic potential, which can be controlled by the cooling rate.

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I. INTRODUCTION

The quasi-one-dimensional conductors have a pair of open sheetlike Fermi surfaces.$^1$ The energy band is given by

$$
e(k_x,k_y,k_z) = -2t_a \cos k_x - 2t_b \cos bk_y - 2t_c \cos ck_z,$$

(1)

where $t_a$, $t_b$, $t_c \approx 3000$ K: 300 K: 10 K $\approx 1: 0.1: 0.03$ in (TMTSF)$_2$ClO$_4$ and (TMTSF)$_2$PF$_6$. Since $t_c$ is very small, we neglect $t_c$ in this paper. In these materials, the field-induced spin-density wave (FISDW) transition$^1$–$^6$ is observed when the magnetic field ($B$) is applied perpendicular to the conductive plane ($a-b$ plane). It has been known$^7$–$^{15}$ that the FISDW is characterized by an integer $N$ of the spin-density wave (SDW) wave number $Q_N = 2k_F + NG$, where $k_F = \frac{\pi}{a}$ is the Fermi wave number and $G = \frac{b + c}{4}$. The state in high field ($B \approx 10$ T) is the SDW state with $N = 0$. The low-field region ($B \approx 10$ T) is divided into the subphases with different $N$. Recently, it is shown that $N$ in the “quantized nesting” is not an integer when the effect of the finite temperatures is considered.$^{16}$ In this paper, we study the FISDW state of (TMTSF)$_2$ClO$_4$ at a very low temperature. Therefore, we consider that the thermal broadening can be neglected and the integer $N$ states are stabilized.

The quantized Hall effect in FISDW states has been observed experimentally, and the sign of the Hall voltage changes in some range of the magnetic field in organic conductors, (TMTSF)$_2$X, $X = $ClO$_4$,$^{6,17,18}$ PF$_6$,$^{19}$–$^{21}$ ReO$_4$,$^{22}$ AsF$_6$,$^{23}$ etc. It has been shown that the quantized value of the Hall conductivity $\sigma_{xy}$ is given by the quantum number of FISDW wave vector, $N$. $^{24}$–$^{26}$ The sign change of the Hall conductivity has been explained by multi-SDW order parameter,$^{27}$ the tight-binding model with the higher transfer integrals ($t_3$ and $t_4$),$^{28}$ or umklapp scattering.$^{29}$

Since the anion ClO$_4$ does not have the inversion center, the orientation of ClO$_4$ on the nearest sites along the $b$ axis aligns in the antiparallel direction below $T_{A0} = 24$ K when (TMTSF)$_2$ClO$_4$ is cooled slowly. It makes the periodic potential

$$\mathcal{H}_V = V \cos\left(\frac{\pi}{b}y\right),$$

(2)

In the presence of the anion ordering, the size of the Brillouin zone becomes half and the energy-band structure is written as

$$\epsilon(k_x,k_y) = -2t_a \cos k_x \pm \sqrt{4t_b^2 \cos^2 bk_y + V^2},$$

(3)

which gives two pairs of open sheetlike Fermi surfaces as seen in Fig. 1.

FIG. 1. The schematic Fermi surface and the FISDW nesting vectors.
The FISDW is strongly affected by the ordering of the anion.30,31 The magnetic field and temperature (B and T) phase diagram in (TMTSF)$_2$ClO$_4$ is different from that in (TMTSF)$_2$PF$_6$, in which there is no anion ordering.36 The unique features in the (TMTSF)$_2$ClO$_4$ are as follows:

(A) The high-field region is occupied by different kinds of SDW subphases: SDW$_1$ (20 T < B < 28 T) and SDW$_2$ (B > 28 T).

(B) There is a first-order transition line between the SDW$_1$ phase and the SDW$_2$ phase.

Lebed and Bak32 first studied the effect of V on the usual FISDW theory as the perturbation for V by assuming V/t$_{bb}$ = 0.05 from 2V/k$_B$T$_{AO}$ < 50 K. They explained the reentrance of the FISDW phase diagram in (TMTSF)$_2$ClO$_4$ at high fields. Osada et al.33 calculated the wave-number dependence of the susceptibility in the perturbation in V as V/t$_{bb}$ = 0.05 and obtained that the peaks of the susceptibility corresponding to the FISDW with even index N are split into smaller double peaks N+ and N−, while the peaks corresponding to FISDW with odd index N are not affected. However, (A) and (B) have not been explained by perturbative calculations32-36 in V.

After these calculations, the strength of the periodic potential has been estimated to be V/t$_{bb}$ = 1.0 ÷ 0.3 from the angle dependence of the magnetoresistance by Yoshino et al.37,38 More recently, Lebed et al.39 have shown a smaller estimation as V/t$_{bb}$ = 0.2. For such a large value of V, the periodic potential should be treated nonperturbatively. The nonperturbative theoretical studies in V have been performed by the self-consistent numerical calculations at T = 0 (Ref. 40) and T ≠ 0,41 the calculations of the noninteracting susceptibility at B ≠ 0 (Ref. 42) and B = 0,43 the Ginzburg-Landau expansion of the free energy at B = 0,44 and the calculations of the random phase approximation at B = 0 (Ref. 45) and B ≠ 0.46 In these nonperturbative calculations, the two pairs of the Fermi sheets are taken account of and all the possible SDW wave vectors Q$_{SDW}$ are studied. When V/t$_{bb}$ = 1.0, it is shown that the coexisting state with Q$_{SDW}$ = (2k$_F$ + 2V/h$_F$, π/2b) and Q$_{SDW}$ = (2k$_F$ - 2V/h$_F$, π/2b) has a lower energy than that of the FISDW state with Q$_{SDW}$ = (2k$_F$, π/b) at high fields. The SDW nesting vectors are shown in Fig. 1. The FISDW state with N = 0 [Q$_{SDW}$ = (2k$_F$, π/b)] has lower energy than FISDW state with N ≠ 0. Then, it has been concluded40-46 that the SDW$_1$ state is the coexisting state with Q$_{SDW}$$^*$$^{(1)}$ = (2k$_F$ + 2V/h$_F$, π/2b) and Q$_{SDW}$$^*$$^{(2)}$ = (2k$_F$ - 2V/h$_F$, π/2b), and the SDW$_2$ state is the FISDW N = 0 state with Q$_{SDW}$ = (2k$_F$, π/b).

Recently, a first-order transition line at low field (7 T < B < 20 T) and low temperature (0.5 K < T < 2 K) has been added to the B and T phase diagram of (TMTSF)$_2$ClO$_4$ under the anion ordering from the experiments of the magnetoresistance.47 (see Fig. 7 of Ref. 47). Haddad et al.48,49 explained the phase diagram in the mean-field theory and the renormalization-group method for large V in nonperturbation.

Very recently, at high fields (from 26 to 45 T), which correspond to the SDW$_2$ state, Uji et al.52 found a periodic oscillation with sign reversal of the Hall resistance (R$_{xy}$). However, the anomalous phenomenon of R$_{xy}$ has not been explained theoretically yet. Uji et al. also observed a periodic oscillation of the magnetoresistance (R$_{xx}$) in the N=1 phase, which is called the rapid oscillations. The rapid oscillations have been observed in many experiments.30,31,47 Although some explanations for the rapid oscillations have been proposed52-54,55-60 for the N=0 phase, the rapid oscillations in the N=1 phase have not been understood yet.

Matsunaga et al.50 have studied the relation between the cooling rate and FISDW in (TMTSF)$_2$ClO$_4$. They found that the B and T phase diagram (see Fig. 4 of Ref. 50) is drastically changed by the cooling rate. Mainly, they showed that the regions of the SDW$_1$ (SDW$_2$) in the B and T plane becomes smaller (larger) as the cooling rate increases. The ordering of anion is expected to be affected by the cooling rate. If the sample is cooled rapidly, the orientation of anion is not ordered and the periodic potential is zero. There may be an effect of the random potential, but we neglect it for simplicity. When the sample is cooled slowly, the orientation of anion will become more rigid depending on the cooling rate. Therefore, we can take the cooling rate as the inverse of the strength (1/V) of the periodic potential.

Moreover, Matsunaga et al.51 observed R$_{xy}$ and R$_{xx}$, in various cooling rates from 0.000 09 K/s (very slow cooling) to 0.33 K/s (rapid cooling) at T = 0.45 and 1.3 K. They found that a negative Hall plateau of R$_{xy}$ appears when the cooling rate is below ~0.03 K/s. In this paper, in order to study the relation between the cooling rate and FISDW of (TMTSF)$_2$ClO$_4$, we focus on the nonperturbative effect of V and the higher harmonics (t$_3$ and t$_4$) of the tight-binding model, because the nonperturbative calculation40-46,48,49 is necessary for large V, and t$_3$ and t$_4$ are thought to be the origin28 of the negative plateau of R$_{xy}$. The parameter V/t$_{bb}$ is changed from 0 to 2. In values such as 0 ≤ V/t$_{bb}$ ≤ 0.2, it has been shown that the peaks of $\chi_0(q)$ at Q$_{SDW}$ and Q$_{SDW}$$^*$ are much smaller than that at Q$_{SDW}$ = 0.40-43 Therefore, the coexistent state with Q$_{SDW}$ and Q$_{SDW}$$^*$ does not have to be considered. We calculate the noninteracting susceptibility [$\chi_0(q)$] near Q$_{SDW}$ by the perturbative and nonperturbative calculations at very low temperature and obtain the 1/V and B phase diagram.

**II. SUSCEPTIBILITY**

We use the linearized dispersion with respect to the $k_x$ and neglect the $k_z$ dependence. The Hamiltonian is given by

$$\hat{H}_0 = \hbar v_F \left( \mp i \frac{\partial}{\partial x} - k_F \right) + t_{\downarrow} \left( - ib \frac{\partial}{\partial y} + Gx \right),$$

(4)

where

$$t_{\downarrow}(k_y) = -2t_b \cos(bk_y) - 2t_0^* \cos(2bk_y) - 2t_3 \cos(3bk_y) - 2t_4 \cos(4bk_y).$$

(5)

The actual values of the higher harmonics (t$^*_0$, t$^*_3$, and t$^*_4$) for (TMTSF)$_2$ClO$_4$ are not known. Osada et al.33 use t$^*_0$/t$_{bb}$ = 0.1 and t$^*_3$ = t$^*_4$ = 0, and Zanchi and Montambaux28 employ 0.03 ≤ t$^*_0$/t$_{bb}$ ≤ 0.2, 0.023 ≤ t$^*_3$/t$_{bb}$ ≤ 0.067, and 0.000 083 ≤ t$^*_4$/t$_{bb}$ ≤ 0.0067. In this paper, for simplicity, we set t$^*_0$/t$_{bb}$ = 10,
\[ \frac{t_2 \nu}{t_3} = 0.1, \quad \frac{t_3 \nu}{t_3} = 0.02, \quad \frac{t_3 \nu}{t_3} = 0.002, \text{ and } k_B T / t_3 = 0.001. \] 
In these parameters, the negative \( N \) for the quantized nesting vector of FISDW is not realized if the periodic potential \( V \) is zero.

First, we consider the susceptibility in the absence of the periodic potential \( V \). We write the eigenstates for the left and right Fermi surfaces in the absence of \( V \) as

\[ \left| (K + nG) \right\rangle = \exp \left[ i \left\{ -k_y + K_n + (nG)x + K_y \right\} \right] \]

and

\[ \left| (K + nG) \right\rangle = \exp \left[ - \frac{1}{h^2 F_G} \int_{0}^{b k_y + G x} t_\perp (p) dp \right] \]

respective, where \( G = (G, 0, 0) = \frac{b \nu B}{k} , 0, 0 \). The eigenvalues for the left and right Fermi surfaces are

\[ \chi_{10}(q) = \sum_{N} \left| J_{10}(q) \right|^2 \chi_{00}(q + NG), \]

where \( \chi_{00}(q) \) is the susceptibility for the one-dimensional (1D) system, which is given by

\[ \chi_{00}(q) = \sum_{K_n} f(E_{K_n + G} - f(E_{K_n + G + q + 2k_F}), \]

\[ = \frac{k_B T}{2 \pi} \int_{-\xi_c}^{\xi_c} \frac{d\xi}{f(-\xi + \epsilon_{q}) - f(-\xi - \epsilon_{q})}, \]

where \( \epsilon_{q} = \frac{k_B T (q - 2k_F)}{2}, \xi_c \) is a cutoff energy of the order of \( t_n \), and \( N(0) \) is the density of states at Fermi energy.

In the presence of the anion potential, the eigenstates for the left and right Fermi surfaces are expressed as the linear combinations of the eigenstates in the absence of \( V \) as

\[ \left| \Psi_{n}^{(0)}(K) \right\rangle = \sum_{m} v_{mn}(\epsilon_{a}) \left| (K + mG) \right\rangle^{(0)}(\epsilon_{a}), \]

where

\[ E_{K_n + G} = -\hbar^2 F_G (K_n + nG), \]

\[ E_{K_n + G}^V = \hbar^2 F_G (K_n + nG), \]

respectively.
\[ [(\mathbf{K} + m\mathbf{G})^{(r)(a)}] = \frac{1}{\sqrt{2}} e^{imbK_x/[h_{\nu F}G]} [(\mathbf{K} + m\mathbf{G})^{(r)}] \]

\[ \pm (-1)^m[(\mathbf{K} + m\mathbf{G} + Q)^{l(r)}] \]

(14)

and \( Q = (0, \pi/b, 0) \). For the states of \([(\mathbf{K} + m\mathbf{G} + Q)^{l(r)}] \), the off-diagonal matrix elements of \( \mathcal{H}_0 + \mathcal{H}_V \) are given by

\[ \langle (\mathbf{K} + m\mathbf{G} + Q)| \mathcal{H}_V| (\mathbf{K} + n\mathbf{G}) \rangle = (-1)^{m-n} V e^{i(m-n) b K_y} \]

\[ \times \sum_{l=-\infty}^{\infty} (-1)^{l} J_{m-n}(3) \left( \frac{4t_b}{h_{\nu F}G} \right) J_1 \left( \frac{4t_3}{3h_{\nu F}G} \right), \]

(15)

\[ \langle (\mathbf{K} + m\mathbf{G} + Q)| \mathcal{H}_V| (\mathbf{K} + n\mathbf{G}) \rangle = V e^{i(m-n) b K_y} \sum_{l=-\infty}^{\infty} J_{m-n}(3) \left( \frac{4t_b}{h_{\nu F}G} \right) J_1 \left( \frac{4t_3}{3h_{\nu F}G} \right), \]

(16)

\[ \langle (\mathbf{K} + m\mathbf{G} + Q)| \mathcal{H}_V| (\mathbf{K} + n\mathbf{G} + Q) \rangle = (-1)^{m-n} V e^{i(m-n) b K_y} \]

\[ \times \sum_{l=-\infty}^{\infty} (-1)^{l} J_{m-n}(3) \left( \frac{4t_b}{h_{\nu F}G} \right) J_1 \left( \frac{4t_3}{3h_{\nu F}G} \right), \]

(17)

By taking \([(\mathbf{K} + m\mathbf{G})^{(r)(a)}] \) as the basis set, we obtain the matrix elements of \( \mathcal{H}_0 + \mathcal{H}_V \) as follows:

\[ (\mathcal{H}_0)^{(a)} = \langle (\mathbf{K} + m\mathbf{G})^{(a)}| \mathcal{H}_0| (\mathbf{K} + n\mathbf{G})^{(a)} \rangle \]

\[ = -\delta_m \cdot h_{\nu F}(K_x + nG), \]

(19)

\[ (\mathcal{H}_0)^{(a)} = (-1)^m V \sum_{l=-\infty}^{\infty} J_{m-n}(3) \left( \frac{4t_b}{h_{\nu F}G} \right) J_1 \left( \frac{4t_3}{3h_{\nu F}G} \right), \]

(20)

\[ (\mathcal{H}_V)^{(a)} = \langle (\mathbf{K} + m\mathbf{G})^{(a)}| \mathcal{H}_V| (\mathbf{K} + n\mathbf{G})^{(a)} \rangle \]

\[ = (-1)^m V \sum_{l=-\infty}^{\infty} J_{m-n}(3) \left( \frac{4t_b}{h_{\nu F}G} \right) J_1 \left( \frac{4t_3}{3h_{\nu F}G} \right). \]

(21)

\[ (\mathcal{H}_V)^{(a)} = \langle (\mathbf{K} + m\mathbf{G})^{(a)}| \mathcal{H}_V| (\mathbf{K} + n\mathbf{G})^{(a)} \rangle \]

\[ = (-1)^m V \sum_{l=-\infty}^{\infty} J_{m-n}(3) \left( \frac{4t_b}{h_{\nu F}G} \right) J_1 \left( \frac{4t_3}{3h_{\nu F}G} \right). \]

(22)

By diagonalizing \( \mathcal{H}_0 + \mathcal{H}_V \) of Eqs. (19)–(22) numerically, \( W^{(a)} \) in Eq. (13) is obtained. The matrix elements of \( \mathcal{H}_0 + \mathcal{H}_V \) in the case of \( t_3 = t_4 = 0 \) have been shown in our previous study.\(^{32}\) The eigenvalues for the left and right Fermi surfaces are

\[ E^{(a)}_{K_x + nG} = -h \nu F(K_x + nG) \pm (-1)^n \Delta, \]

(23)

\[ E^{(a)}_{K_x + nG} = h \nu F(K_x + nG) \pm (-1)^n \Delta, \]

(24)

where \( \Delta \) is the splitting of the energy band. The eigenvalue structure under the magnetic field is shown in Fig. 2. In the case of perturbation\(^{33}\) in \( V \), the eigenvalue is obtained by \( E^{(a)}_{K_x + nG} = \frac{1}{2}[(\mathbf{K})^{(r)(a)}] + [(\mathbf{Q})^{(r)(a)}] \) of Eq. (14) and the energy splitting becomes

\[ \Delta_0 = \sum_{l=-\infty}^{\infty} V J_{l-3} \left( \frac{B_0}{B} \right) J_1 \left( \frac{t_3 B_0}{3t_3 B} \right), \]

(25)

where \( B_0 = \frac{4t_b}{h_{\nu F}G} \). When we set \( k_F = \frac{\pi}{4a} \) (quarter-filled band), \( t_b / t_3 = 0.1 \), and \( a = b = 0.7 \) nm, we get \( B_0 \approx 400 \) T. It is found that \( \Delta_0 \) depends on \( B \), \( V \), \( t_b \), and \( t_3 \). When \( \nu F G \) in Eq. (25) is very small, \( \Delta_0 \approx V J_0 \left( \frac{B_0}{B} \right) \). On the other hand, when the calculation is performed in the nonperturbation\(^{42}\) in \( V \), it has been shown that \( \Delta \neq \Delta_0 \) at \( V = 0.1 \). In this case, \( \Delta \) depends on \( B \) and all band parameters such as \( V, t_b, t_3, t_4 \), and \( t_3 \), because we obtain the eigenvalue by diagonalizing the Hamiltonian numerically.

The susceptibility is obtained as
The susceptibility for the nonperturbative calculation is given by

\[
\chi_0(q) = \sum_{n,n'} \sum_{\mathbf{K}} \left| \langle \Psi_{n'}^{(n)}(\mathbf{K} + \mathbf{q} - 2\mathbf{k}_F) | \mathbf{e}^{i\mathbf{q} \cdot \mathbf{r}} | \Psi_n^{(n)}(\mathbf{K}) \rangle \right|^2 \times \frac{f(E_{K_n+G}(q)) - f(E_{K_n+G-2\mathbf{k}_F+2nG})}{E_{K_n+G-2\mathbf{k}_F+2nG} - E_{K_n+G}},
\]

or\( (26) \)

Then, the susceptibility for the nonperturbative calculation is given by\(^{33}\)

\[
\chi_0(q) = \sum_n \left[ |A_{2n+1}^{+}(q)_n|^2 \chi_0^{+}(q - 2\mathbf{k}_F + 2nG) + |A_{2n}^{-}(q)_n|^2 \chi_0^{-}(q - 2\mathbf{k}_F + 2nG) + |A_{2n}^{+}(q)_n|^2 \chi_0^{0}(q - 2\mathbf{k}_F + (2n + 1)G) \right],
\]

or\( (27) \)

where

\[
A_{2n+1}^{+}(q)_n = \sum_{m,m'} v_{m_0}^{(n)+} v_{m'}^{(n)+} e^{i(m-m')\mathbf{q} / 2} I_{m-m'+2n}(q),
\]

or\( (28) \)

\[
A_{2n}^{-}(q)_n = \sum_{m,m'} (-1)^{m+m'} v_{m_0}^{(n)+} v_{m'}^{(n)+} e^{i(m-m')\mathbf{q} / 2} I_{m-m'+2n}(q),
\]

or\( (29) \)

\[
A_{2n+1}^{-}(q)_n = \sum_{m,m'} (-1)^{m+m'} v_{m_0}^{(n)+} v_{m'}^{(n)+} e^{i(m-m')\mathbf{q} / 2} I_{m-m'+2n+1}(q),
\]

or\( (30) \)

\[
A_{2n}^{+}(q)_n = \sum_{m,m'} (-1)^{m+m'} v_{m_0}^{(n)+} v_{m'}^{(n)+} e^{i(m-m')\mathbf{q} / 2} I_{m-m'+2n+1}(q),
\]

or\( (31) \)

and

For the perturbational calculation, the susceptibility is simply given by

\[
\chi_0(q) = \sum_{n= \infty}^{\infty} \sum_{\mathbf{K}} \left[ |I_{2n}(q)|^2 \frac{f(E_{K_n+G}^{(n)+}) - f(E_{K_n+G-2\mathbf{k}_F+2nG}^{(n)+})}{E_{K_n+G-2\mathbf{k}_F+2nG}^{(n)+} - E_{K_n+G}^{(n)+}} + |I_{2n}(q)|^2 \frac{f(E_{K_n+G}^{(n)-}) - f(E_{K_n+G-2\mathbf{k}_F+2nG}^{(n)-})}{E_{K_n+G-2\mathbf{k}_F+2nG}^{(n)-} - E_{K_n+G}^{(n)-}} + |I_{2n+1}(q)|^2 \frac{f(E_{K_n+G}^{(n)+}) - f(E_{K_n+G-2\mathbf{k}_F+(2n+1)G}^{(n)+})}{E_{K_n+G-2\mathbf{k}_F+(2n+1)G}^{(n)+} - E_{K_n+G}^{(n)+}} + |I_{2n+1}(q)|^2 \frac{f(E_{K_n+G}^{(n)-}) - f(E_{K_n+G-2\mathbf{k}_F+(2n+1)G}^{(n)-})}{E_{K_n+G-2\mathbf{k}_F+(2n+1)G}^{(n)-} - E_{K_n+G}^{(n)-}} \right],
\]

or\( (35) \)

where \( E_{K_n+G}^{(n)+} = -\hbar v_F (K_n + nG) \pm (1)^n \Delta_0 \) and \( E_{K_n+G}^{(n)-} = -\hbar v_F (K_n + nG) \pm (1)^n \Delta_0 \). Osada et al.\(^{33}\) have shown Eq.\( (35) \) in the case of \( t_3 = t_4 = 0 \).

\section{III. RESULTS AND DISCUSSIONS}

The susceptibility \( \chi_0(q) \) in the cases of the nonperturbation and the perturbation is calculated numerically by using Eqs.\( (27) \) and \( (35) \), respectively. For \( 1/(V/t_3) = 5 \) and \( B/B_0 = 0.02889 \), \( \chi_0(q) \) for the nonperturbative calculation is shown in Figs.\ 3 and 4, where \( 2\Delta/(\hbar v_F G) \) is \(-0.40095\). In this case, \( 2\pm \) means \( q = 2k_F + NG \pm 2\Delta/\hbar v_F = 2k_F \)
value is very small, the splitting of the peaks of $\Delta = 33.33$ and $10$ in Figs. 10 and 11, respectively. $1/N - 2$ are highest. Thus, in this parameter, the negative $N$ is little. It is found from Fig. 6 that the peaks of $-2+$ and $-2-$ are almost the same. Therefore, we show only values for $V/t_b = 0.03$ and $0.1$, where $2$ is dominant. $42$ Therefore, in the region in which the perturbation is justified, we can understand the SDW nesting from the picture by using only one term of $\psi^{(\sigma)}_{m}$. In fact, Osada et al. explained the $B$ dependence of $\chi_0(q)$ of even (odd) $N$ from the picture of the intra(subband) pairings, respectively, as seen in Fig. 2. The SDW nesting of odd $N$ such as $N = 1, 3, \ldots$, makes the intersubband pairing, where the SDW $N = 1$ nesting is $|K|$.
Therefore, although it is difficult to explain the oscillation of $\chi_0(q)$, respectively, as seen in Fig. 2. In the case of the intersubband pairing, the SDW nesting vector is not influenced even if $\Delta$ is oscillated as a function of $B$. Thus, $\chi_0(q)$ is unchanged. On the other hand, the SDW nesting of even $N$ such as $N=0\pm,2\pm,\ldots$, gives the intrasubband pairing (for example, the SDW nesting of $N=0+$ and $N=0-$ are $(K_r^{(\pm)})\rightarrow|\langle K_r^{(\pm)}\rangle|$, denoted by a solid arrow, and $(K_r^{(\pm)})\rightarrow|\langle K_r^{(\pm)}\rangle|$, denoted by a dotted arrow, respectively, as seen in Fig. 2). As a result, the SDW nesting vectors and $\chi_0(q)$ are oscillated due to the oscillation of $\Delta$ when $B$ is changed. However, this picture cannot be applied to the nonperturbation, because the eigenstate of Eq. (13) is not described by the single state of $|\langle K_r^{(\pm)}\rangle|$. Therefore, although it is difficult to explain the oscillation of $\chi_0(q)$ of odd $N$ in the nonperturbation by the simple picture, it may be an explanation of the experimentally observed rapid oscillation of $N=1$ phase.\(^{30,31,47}\)

Using the highest peaks of $\chi_0(q)$ at $0.001 \leq V/t_0 \leq 0.2$ [5 $\leq 1/(V/t_0) \leq 1000$] (for example, in Fig. 8, the peaks for negative $N=-2^+$ are the largest near $B/B_0=0.023$), we obtain the $1/V$ vs $B$ phase diagrams for nonperturbative and perturbative calculations in Fig. 12. We write even $N$ as even $N$, because the peak heights of $N^+$ and $N^-$ are almost the same. At large $V [1/(V/t_b) \leq 33.33$, that is, $V/t_b \geq 0.03]$, $N=-2$ and $N=2$ phases exist in $N=3$ phase only in the nonperturbative calculation. This difference comes from the field dependence of the local peaks of $\chi_0(q)$; that is, the phases of the oscillation of the local peaks for even $N$ are opposite to those for odd $N$ in the nonperturbative calculation, which can be seen in Figs. 7−9. In the $1/V$ and $B$ phase diagram of the nonperturbation, the negative ($N=-2$) phase appears when $1/(V/t_b) \approx 30$, whose features qualitatively agree with the recent experimental results by Matsunaga et al.\(^{51}\)

Zanchi and Montambaux\(^{28}\) have shown that the higher harmonics ($t_3$ and $t_4$) are necessary for the existence of the negative $N$ phase in the absence of anion ordering. In order to see whether $t_3$ and $t_4$ are necessary for the negative $N$ phase in the presence of anion ordering, the local maximum values of $\chi_0(q)$ at $t_3=t_4=0$ and $V/t_b=0.1$ in nonperturbation are shown in Fig. 13. There is no peak of negative $N$. When the values of $t_3$ and $t_4$ are finite, the peak of $N=-2^+$ at $B/B_0 \approx 0.023$ becomes larger than that of $N=4^+$, as seen in Fig. 8. Thus, $t_3$ and $t_4$ are needed for the appearance of the negative $N=-2$ phase even if anion ordering exists.

**IV. CONCLUSION**

We have studied the effects of the strength of the periodic potential $V$ on the phases of FISDW in quasi-one-dimensional conductors. We have calculated the local peaks of the susceptibility as a function of the magnetic field $B$ for various values of $V$ and obtained the $1/V$ and $B$ phase diagrams. We have shown that in ClO$_4$ the negative phase ($N=-2$) exists for large values of $V (V/t_b \approx 0.03)$, even for small values of the higher-order hoppings, $t_3$ and $t_4$, where there exists only positive $N$ in the absence of $V$. The obtained phase diagram is in good agreement with that reported recently by Matsunaga et al.\(^{51}\) For the large value...
of $V$, the peaks of $\chi_0(\mathbf{q})$ of odd $N$ oscillate as a function of $B$ in the opposite phase to the peaks of even $N$. Thus the FISDW with $N=-2$, which is suppressed by $V$ in general, is stabilized in some region in the $1/V$ and $B$ phase diagram, because the FISDWs with odd $N$ are much suppressed in these regions of parameters.

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